Identification of Non-Gaussian distributed models from pulsed radar returns

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***Abstract*—In coastal surveillance pulsed radar system, employing a non-coherent receiver, the radar returns consist of information received from the targets, along with unwanted signals like receiver noise and clutter. In non-moving target indicator (MTI) pulsed radars, the target information is in the form of amplitude which depends on the radar cross section (RCS) of the target. Practically it has been observed that the amplitude statistics of the mean sea level decays exponentially. Thus, target along with receiver noise follows Gaussian distribution. In coastal surveillance scenario, the clutter usually consists of returns from sea surface, rain, land, etc. Hence, by employing the conventional constant false alarm (CFAR) detectors, probability of false alarms will be more in the scenarios like target presence along with clutter (sea spikes, rain, etc). Usually, the probability distributions of the amplitude returns from clutter will follow non-Gaussian models such as Weibull, Log-normal, K-distribution, etc. But the reflection from sea occurs due to many small scatterers having relative motion with each other. They appear as discrete spikes, which can interfere with the target reflection. Hence, the subtraction of mean sea level amplitude from the amplitude of the radar returns is not always sufficient to differentiate a target from the sea clutter. The presence of discrete spikes increases the skewness of the distribution. Thus, classifying the radar returns by fitting it into Gaussian and non-Gaussian model can help us to differentiate between returns consisting of only target with noise, or target along with noise and clutter respectively. In this paper we have proposed a method to find the rate of convergence of the probability distribution function (PDF) by computing the cumulative distribution function (CDF) on a two dimensional radar data. It is shown that Extreme Value Theory (EVT) can be used to approximate the tail of the underlying clutter distribution. This method can be used to identify the presence of clutter i.e. non-Gaussian model and further EVT can be used to calculate the detection threshold which is independent of the underlying distribution.**

Keywords: pulsed radar, Gaussian and non-Gaussian model, clutter distribution, Extreme Value Theory.

I. Introduction

In coastal surveillance scenario, sea clutter acts as a major hindrance in detection of the targets. Sea clutter depends on the various environmental factors like roughness of the sea, wind speed, sea state, etc. Thus, sea clutter is a purely random and stochastic process.

Practically, it has been observed that the radar returns consisting only of target and receiver noise follows Gaussian distribution. Radar returns consisting of sea clutter, rains, etc. are generally non-Gaussian in nature. The commonly used CFAR techniques are CA-CFAR, GO-CFAR, SO-CFAR, OS-CFAR [1],[2]. These methods assumes the nature of the radar clutter returns as Gaussian in nature. The performance of CA-CFAR degrades as the environment becomes heterogeneous. OS-CFAR works well in heterogeneous environment but its efficiency degrades in homogeneous environment. Thus, these CFAR detectors are application specific. But in real time scenario, due to the presence of clutter the Gaussian nature of the background environment consisting of sea clutter changes. Understanding the non-Gaussian nature of the sea clutter depends on the amplitude distribution of the radar returns. Thus, there is a need to distinguish between Gaussian and non-Gaussian radar returns and then adaptively set the detection threshold based on the environmental conditions.

According to the Extreme Value Theory (EVT), the existing tail of any distribution function can be modelled using the generalized pareto distribution (GPD) [3]. Non-Gaussian distribution for e.g. Rayleigh, Log-normal, Weibull distribution have large tail in their probability distribution function, which can be used to distinguish it from Gaussian distribution [4]. Thus, using the principles of Extreme Value Theory, the tail of the radar clutter returns can be approximated to GPD and hence, a distribution independent CFAR can be designed.

II. Extreme Value Theory

Extreme value theory states that as the number of samples increases, the distribution of the maximum value of identically independent random variables can be expressed as [5]:

$$H(x) = \begin{cases} 1 - e^{-(1+\xi x)^{-1/\xi}} & \xi \neq 0, (1+\xi x) > 0 \\ 1 - e^{-e^{x}} & \xi = 0, x \in \mathcal{R} \end{cases} \quad (1)$$

where ξ is the shape parameter.

If (1) holds, the conditional probability distribution $F_{T_g}(x) = P(X - T_g \leq x \mid X > T_g)$ converges to GPD. The cumulative distribution function of the GPD is defined by [6]:

$$G(x \mid \beta, \xi) = \begin{cases} 1 - (1 + \xi \frac{x}{\beta})^{-1/\xi} & \xi \neq 0, (1 + \xi \frac{x}{\beta)} > 0 \\ 1 - e^{-x/\beta} & \xi = 0, x \in \mathcal{R} \end{cases} \tag{2}$$

where ξ is the same shape parameter as in (1), β is the scale parameter, and T_g is the threshold chosen from the ordered statistics (tail part of the probability density fucntion) of the reference window. This can be used to model the tails of distribution, i.e. for data exceeding certain threshold. It has been observed that the distributions such as Weibull, Rayleigh, Log normal, etc which have visible tail in their distribution can be characterized by GPD.

As the value of ξ increases, the skewness of the tail increases. Thus, this ξ can be used to distinguish between Gaussian and non-Gaussian distribution [4].

Generally, targets have strong backscattering returns than the clutter; thus, the targets have greater amplitude returns with low probablity of occurence. Hence, targets contribute to the tail part of the histogram [7]. Thus, this histogram or the computed probability density function can be used for first thresholding the ordered statistics of the CFAR window under analysis.

III. CFAR Detector and Algorithm Description

This algorithm is based on two steps threshold calculation. First step is the two-dimensional histogram analysis of the amplitude returns of a particular area with respect to the desired number of azimuth count pulse and range bins. This analysis will give the tail part of the histogram. Second step is the calculation of detection threshold based on the EVT CFAR detector.

A. Computing first threshold based on the two dimensional histogram analysis:

Choose a two dimensional window consisting of the N number of cells. The non-zero samples are then used to calculate the histogram or the probability density function.

The tail part of the computed density function of the radar returns consists of target which are generally of low probability of occurrence with high amplitude [7]. Let F be the cumulative distribution function obtained from the probability density function of the area under the analysis.

$$F(T_g) = \phi \tag{3}$$

where, T_g is the first local threshold and ϕ indicates the desired percentage of data beyond which the probability of occurence of target is high. Thus if ϕ is known, T_g can be found out easily. Amplitude value of the radar returns smaller than the T_g can easily be ignored. This threshold marks the beginning of the tail, which we can fit into GPD function. Let the total number of radar returns greater than the first threshold be M. Thus, the tail probability is:

$$P_{T_g} = P(X > T_g) = M/N \tag{4}$$

B. Computing second threshold based on the EVT:

The first threshold T_g must be much smaller than the detection threshold to fit the GPD in the tail of the histogram. The remaining M number of cells is used to approximate the tail distribution; i.e. to fit the tail in G. To fit the curve in GPD, take only the exceedances of the remaining samples i.e. subtract the first threshold value from the remaining M samples.

$$y(i) = x(i) - T_g \quad ; \quad i = 1...M \tag{5}$$

Thus, $y(i)$ represents the value of exceedances over the threshold T_g, provided this threshold has been exceeded. These exceedances value can be used to approximate the tail of the distribution with GPD [8]:

$$F_{T_g}(y) = G(y \mid \xi, \beta) \quad ; \quad y = x - T_g \tag{6}$$

The final detection threshold T_d is such that:

$$\alpha = P(X > T_d) = P_{T_g}.P_w \tag{7}$$

where $P_w = P(y > w) = 1 - G(y \mid \xi, \beta)$ and α is the probability of the value exceeding the detection threshold T_d. The parameters ξ and β of the GPD can be found out using the maximum likelihood function. These parameter values are used to calculate the final detection threshold, T_d [9]:

$$T_d = T_g + \frac{\beta}{\xi}\big[(\alpha\frac{N}{M})^{-\xi} - 1\big] \tag{8}$$

Thus, this detection threshold, T_d will keep on adapting according to the changing environmental conditions, while keeping α constant every time. The value of the cell under test (CUT) is compared with T_d. The decision is realized by simple thresholding:

$$x = \begin{cases} \text{Target} & x \geq T_d \\ \text{No Target} & x < T_d \end{cases} \tag{9}$$

where, x is the amplitude of CUT. Hence, this CFAR is independent of the underlying clutter distribution. This method takes in consideration only the tail part of the histogram of the radar returns. So instead of processing whole data of the window under consideration, this method censors most of the data by histogram based first thresholding. In this way, the total computational load reduces.

IV. Results and Analysis

Based on the model discussed in the previous section, few simulations were carried out on data consisting of target and sea clutter radar returns. The data is obtained from a commercial navigation S-band radar. It has a range resolution of 35 meters. The operating frequency of the radar is 3050 MHz. It has 4096 azimuth count pulse with 25 rpm. It has been observed that more the number of data cells under the observation window the better is the performance of the CFAR detector.

A. *Case 1: Analysis area having sea clutter and target*

Fig.1 shows the polar plot of the radar data returns consisting of sea clutter, cloud and targets. The top right of the analysis area contains strong sea spikes. This is a case of heterogeneous environment. Fig.2 shows the computed density function of the analysis area marked in fig.1. Using this density function, amplitude of data having cumulative probability, ϕ as 0.70 has been taken as the first threshold, T_g. The calculated value of T_g is 152. The data exceeding the first threshold, T_g is used for the GPD parameter calculations. The calculated values of ξ and β are 0.3718 and 25.2806 respectively. The value of ξ indicates the existence of heavy tail. Thus, this is a non-Gaussian distributed model. Hence, EVT is applicable here. Using (8), the calculated value of detection threshold, T_d is 193.7422.

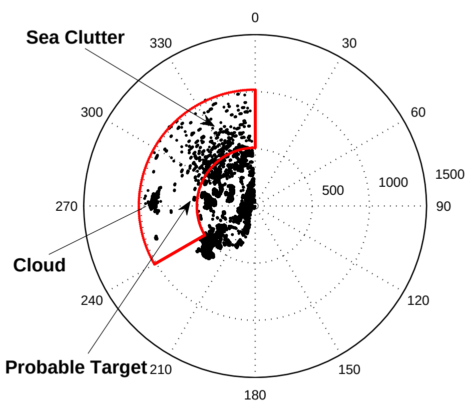

Fig. 1. Polar plot of radar returns with analysis area having sea clutter and target.

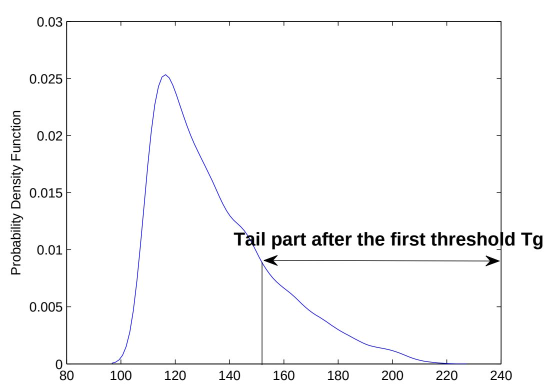

Fig. 2. Probability density function of the area marked in fig 1. The tail is clearly visible.

Fig.3 shows the radar returns of fig.1 with suppressed clutter after applying the detection threshold. With EVT CFAR detector, the targets are retained. Sea clutter with strong spikes and cloud are suppressed with few false alarms. We can further reduce these false alarms with scan-to-scan correlation.

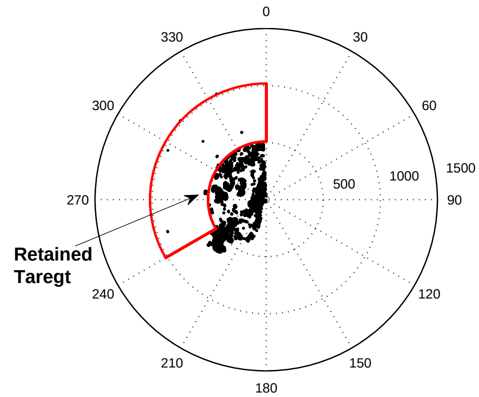

Fig. 3. Polar plot of the radar returns of fig.1 after applying CFAR using GPD.

Fig.4 shows A-scope of a particular range bin of the analysis area of fig.1. Here, the receiver noise can be eliminated easily using a hard threshold. The amplitude level above straight line is the retained target using EVT CFAR. This method eliminates the strong sea spikes. The dotted line shows the threshold calculated using conventional CA CFAR of window length 32. This method also retains the target. But it fails in area where sea spikes are present. Here, the calculated threshold is below the sea spikes. Hence, CA CFAR is prone to false alarms in heterogeneous clutter environment.

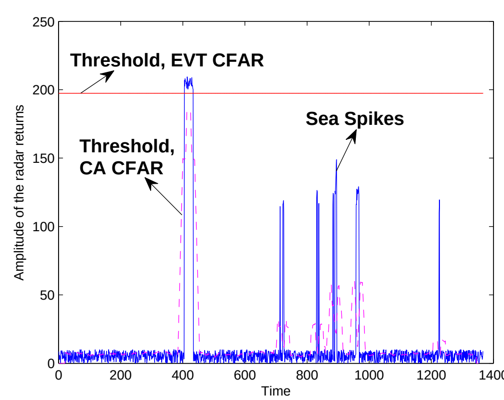

Fig. 4. Amplitude versus time in a particular range bin of the analysis area of fig.1. The straight line shows the threshold, T_d using EVT CFAR. The dotted line shows the threshold calculated using CA CFAR.

B. *Case 2: Analysis area having clutter and target with small window length*

Fig.5 shows polar plot with analysis area having target and land clutter. As shown in fig.6, when this case is passed through EVT CFAR detector, we can detect the target easily. Since, the analysis area is having clutter like strong land patch, it is difficult to fully suppress the clutter. Land clutter further breaks into small patch and can produce false alarms. Thus, this can be considered as the drawback of the GPD.

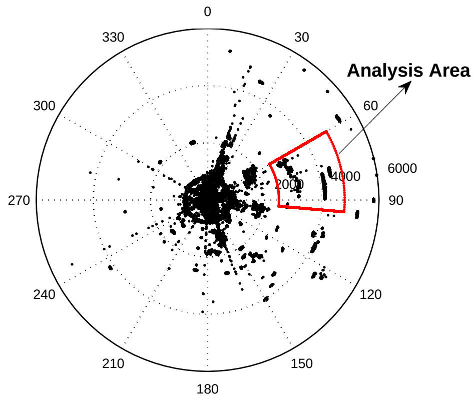

Fig. 5. Polar plot of radar returns with analysis area containing target and land clutter.

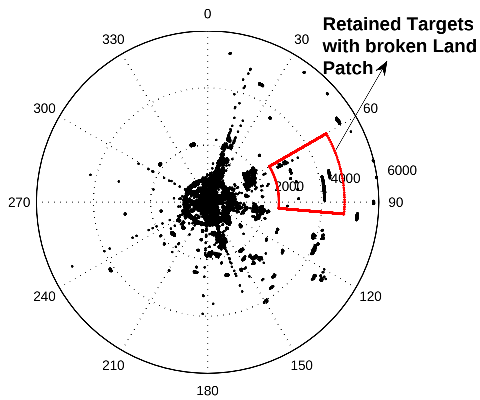

Fig. 6. Polar plot of the radar returns of fig.5 after applying CFAR using GPD.

C. Case 3: Analysis area having clutter and target with large window length

Fig.7 shows polar plot taking analysis area stretched all over. Fig.8 shows the final scenario of fig.7 with suppressed clutter. It has been observed that with this method we can retain the targets. Since, the window length of the CFAR detector is large, the presence of land clutter does not degrades the performance as in case 2. Thus, larger the length of window better is the performance of the EVT-CFAR detector.

V. Conclusion

The aforementioned method shows that GPD can be used to approximate the tail of the underlying clutter distribution in a sea clutter scenario, which can be used to distinguish between Gaussian and non-Gaussian distributed models. Further, it can be used to devise distribution independent CFAR detector. The more the number of cells in the CFAR window, the better is the effectiveness of this method. Thus, as shown this method

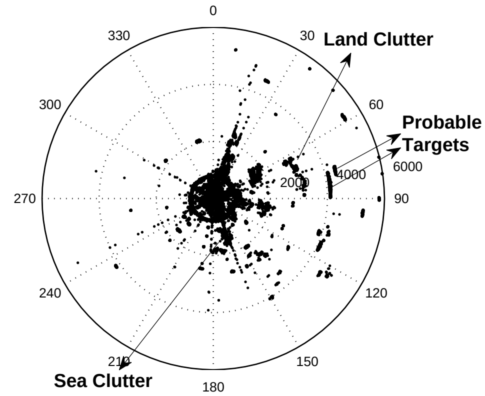

Fig. 7. Polar plot of radar returns with analysis area spread over larger distance.

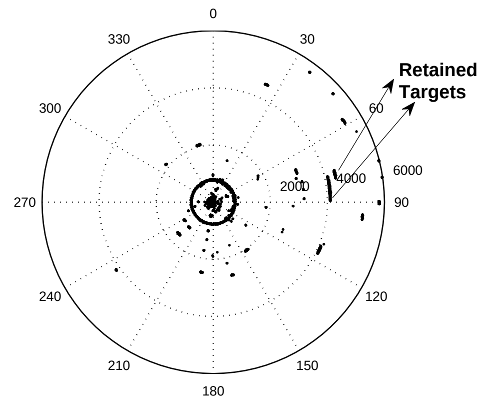

Fig. 8. Polar plot of the radar returns of fig.7 after applying CFAR using GPD.

can be used more effectively as compared to CA-CFAR in heterogeneous clutter scenario.

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References

[1] Hermann Rohling, AEG-Telefunken, Radar CFAR Thresholding in Clutter and Multiple Target Situations,IEEE Transactions on Aerospace and Electronic Systems Vol. AES-19, No. 4 July 1983.

[2] Gandhi P.P., Kassam S.A., Analysis of CFAR Processors in Nonhomogeneous Background, IEEE Transactions on Aerospace and Electronic Systems, vol. 24, no. 4, July 1988, pp. 427-445.

[3] P. Embrechts, C. Kluppelberg, T. Mikosch, Modeling Extremal Events for Insurance and Finance, Berlin: Springer-Verlag, 1997.

[4] Burnecki K, Wylomanska A, Chechkin A (2015) Discriminating between Light- and Heavy-Tailed Distributions with Limit Theorem. PLoS ONE 10(12): e0145604.

[5] J. Pickands, Statistical inference using extreme order statistics, Annals of Statistics, vol. 3, 1975, pp. 119-131.

[6] www.wikipedia.org/wiki/Generalized Pareto distribution.

[7] Gui Gao, Li Liu, Lingjun Zhao, Gongtao Shi, and Gangyao Kuang, An adaptive and fast CFAR Algorithm Based on Automatic Censoring for Target Detection in High-Resolution SAR Images, Transactions on Geoscience and Remote Sensing, Vol 47, No 6, June 2009.

[8] Micha Piotrkowski, An extreme value theory approach to correlated radar clutter, Institute of Electronic Systems, Warsaw University of Tecnology, 2007.

[9] McNeil U Frey, Estimation of tail-related risk measures for heteroskedastic financial time series: an extreme value approach, Journal of empirical finance, 2000.

[10] G. Gao, G. Kuang, Q. Zhang, and D. Li, Fast detecting and locating groups of targets in high-resolution SAR images, Pattern Recognition., vol. 40, no. 4, pp. 1378-1384, Apr. 2007.

[11] M.I. Skolnik. Introduction to Radar System (3nd, ed.). McGraw-Hill, New York, 2001.